Household Stock Market Participation and Exit:
The Role of Homeownership

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Abstract

This paper argues that a large part of the stock market participation puzzle is driven by high stock market exit rates among participants: In the US, 20% of households who have stock hold no stocks two years later. Using survey data I show that stock market exit frequently coincides with renting households becoming first-time owners. After estimating a life-cycle model of portfolio choice with housing and per-period participation costs, I show that it quantitatively matches the US participation rate, homeownership rate, and entry/exit in stock markets over the entire life-cycle. The introduction of housing increases the exit rate among young new homeowners and reduces the participation rate among middle-aged and retired households by decreasing liquid wealth. Housing reduces the unexplained participation gap between the model and the data by 71%, compared to a model without housing.

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1 Introduction

Only half of US households hold any stocks or mutual funds at any given time. Among current stock market participants, 20% exit the stock market and hold no stocks two years later. At the same time, the majority of US households own their primary residence. Given the high equity premium, the low stock market participation rate challenges both economic theory and retirement policy that relies on households’ private savings. In this paper, I ask whether homeownership and the frictions associated with housing can explain both the low stock market rate and the high exit rate among participants.

I first document the high exit rate from stock markets among US households. At any age, the two-year exit rate is between 15-40%, while the entry rate ranges from 8-16%. This novel fact underscores that the low participation rate is driven by both low entry rates and high exit rates. Second, I show that homeownership is associated with non-participation. All else equal, the stock market participation rate of owners is six percentage points lower than that of renters. Third, the participation dynamics differ by house tenure: the exit rate is substantially higher among renters, and becoming a homeowner increases the probability of exit by 16 percentage points, all else equal. These facts suggest that a life-cycle model of stock market participation should take into account the joint participation-ownership dynamics.

Motivated by these stylized facts, I extend the workhorse life-cycle portfolio choice model in Cocco, Gomes, and Maenhout (2005). The extended model adds a per-period participation cost and house-tenure decisions to a realistic life-cycle model and can jointly account for the participation rate, homeownership rate, conditional portfolio weight on stocks, and net worth over the entire life cycle. To do so, the model requires high participation costs ($199 per year, in 1995 dollars) and a high constant relative risk aversion parameter ($\gamma = 12.36$). My model also nests Fagereng, Gottlieb, and Guiso (2017) who study limited stock market participation in Norway.

\footnote{Author’s calculation from the Panel Study of Income Dynamics, see section 2 for details.}
but omit housing. To understand the contribution of housing to the participation puzzle, I estimate the model with and without housing. Housing drives down participation mainly by decreasing homeowners’ liquid wealth. Owners prefer to build home equity to move away from the mortgage borrowing constraint before investing in stocks. Secondly, since renters aspire to become homeowners, they have short investment time horizons, which decrease the attractiveness of the stock market and drive down their participation rates. As in the data, the model shows that renters who become homeowners often liquidate their stock holdings to finance a down payment.

To test the validity of the model, I perform three exercises. First, the model matches the entry and exit rates into the stock market over the life cycle, even though this moment is not targeted in the estimation. Second, the model generates lower participation rates among renters than homeowners, a key feature of the data. Third, Chetty, Sándor, and Szeidl (2017) obtain causal estimates of the effects of home equity and mortgage debt on the portfolio weights of US households. Using the same empirical models on the model-simulated panel provides estimates that are similar, showing that the model not only matches aggregate moments but also the causal effects of housing on portfolio choices in the US.

To understand the interplay between homeownership, financial frictions, and stock market participation, I perform a policy experiment where the down payment increases. An increase in the down payment delays homeownership. By delaying homeownership, the participation rate of renters goes up, as they hold more liquid wealth. At the same time, this means that the fraction of middle-aged households with low liquid increases, decreasing the participation rate of middle-aged households. Finally, I study the relative importance of the three ex-ante sources of risk (income, house prices, and stock returns) households face in the model and find that income risk/inequality is the main source of wealth dispersion. Still, the calculation shows that house price risk at age 60 causes the 20th percentile of the wealth
distribution to have 2/3 lower net worth than the 80th percentile.

1.1 Relation to the Existing Literature

The main contributions of this paper is to demonstrate

1. That high exit rates among participating households is an important driver behind limited stock market participation.

2. That house tenure changes are associated with stock market exit.

3. That a model with rent/own choice and participation decisions can explain the stock market participation rate and the high exit rate.

I provide a general literature review before discussing the two papers that are closest to mine.

There is a large body of literature on the life-cycle profiles of portfolio choices in models with realistic income processes (e.g., Cocco et al. (2005); Chang, Hong, and Karabarbounis (2018); Wachter and Yogo (2010)). This literature generally focuses on the portfolio weights and ignores the participation decision. One strand of this literature, owing back to Vissering-Jørgensen (2002) focuses specifically on limited stock market participation (Cocco (2005); Calvet, Campbell, and Sodini (2007); Athreya, Ionescu, and Neelakantan (2017); Catherine (2019)), who respectively use a one-time fixed cost, investor mistakes, costly human capital investment or correlations between stock returns and income as mechanisms to explain non-participation. Cocco (2005) and Yao and Zhang (2005) study portfolio choices in the presence of housing, but each paper omits either extensive margin, house tenure or the participation decision, respectively. My paper contributes to our understanding of the stock market participation puzzle by showing that a satisfying theory of limited stock market participation should also explain the high exit rate and that illiquid housing does
Some empirical papers have cast doubt on the theoretical findings from life-cycle models with housing and stock market portfolio decisions, but Chetty et al. (2017) reconciles theory with evidence and finds that home equity or property values have large effects on portfolio weights and participation decisions. Further, Beaubrun-Diant and Maury (2016) in an empirical paper explicitly consider the joint tenure-participation decisions of US households and find that the two decisions are interdependent. My paper contributes by showing that housing is important for participation decisions and, in particular, for stock market exit.

Fagereng et al. (2017) use high-quality Norwegian administrative data to document a dual re-balancing away from stocks around retirement: households first decrease portfolio weights as they age, and then gradually exit the stock market. After extending the workhorse model by Cocco et al. (2005) to include a per-period participation cost and stock market tail events, Fagereng et al. (2017) show that the estimated model can match the dual adjustment. This age-dependent interaction between a participation cost and tail events is novel and occurs because older households are more reliant on financial assets, and thus stock market crashes are more costly, and they hold lower portfolio shares than younger households. However, the model cannot generate sufficiently limited stock market participation, with over 90% of middle-aged households participating versus the 55% participating in the data. My paper contributes by showing that the addition of owner-occupied housing into this framework can jointly explain the low participation rate over the entire life cycle, portfolio weights, and the high exit rate.

While Fagereng et al. (2017) omits housing, Vestman (2019) focuses on the large difference in the participation rate among renters and homeowners. Using Swedish

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2Briggs, Cesarini, Lindqvist, and Östling (2020) use Swedish register data and show that households do not immediately enter the stock market even when winning large lotteries. This is hard to explain solely with reasonable one-time fixed costs or per-period costs. In line with my model, they find that the treatment effects on participation is smaller among renters.
data, he argues that housing seems relatively unimportant in explaining the participation decision among Swedish households but finds evidence for unobserved household fixed effects. He then builds a model where households face a one-time participation cost and a house tenure decision. To match a peak participation rate of 75%, he uses preference heterogeneity, where one type (which tends to own) is more patient, has lower risk aversion, and faces lower participation costs than the other type (which tends to rent). This preference heterogeneity can match the participation gap between renters and owners, a non-trivial achievement since renters have more to gain from participation. Vestman’s work is the first to study the house tenure decision together with the entry decision. However, the calibrated model counter-factually predicts 100% participation among retirees and overpredicts participation at all ages above 35. Moreover, a model without per-period participation costs cannot generate exit from stock markets.

Compared to these two papers, this paper contains several innovations. First, I take the model to US data, where institutional differences make private savings more important for consumption after retirement. Second, the combination of housing and per-period cost allows the model to qualitatively and quantitatively match the participation rate and other moments over the life-cycle. I show that per-period costs combined with tail-events can lower participation rates among renters. The mechanism behind this result is similar to the dual adjustment in Fagereng et al. (2017): renters need sufficient financial wealth for a down payment and are less likely to participate in the stock market than owners, all else equal. I compliment the work in Vestman (2019) by highlighting another mechanism (tail events and per-period costs together with borrowing constraints) that induces a participation gap between owners and renters. Further, homeowners with low home equity and low liquid wealth (‘home poor’) prefer to build home equity to stock investment, decreasing participation even among the middle-aged.
2 Data

The Panel Study of Income Dynamics (PSID) is a longitudinal household survey compiled by the University of Michigan since 1968. Since the PSID follows individuals and households over time, we can study household participation and portfolio decisions preceding, during, and after house tenure changes. From 1999 detailed wealth information has been collected bi-yearly. The largest drawback of the PSID is that the household’s financial portfolio allocations are poorly measured.

To complement the PSID and obtain more accurate portfolio information I also use the Survey of Consumer Finances (SCF). The SCF is a cross-sectional survey of US households and is conducted every three years by the Federal Reserve Board. The main advantage of the SCF is that it includes more detailed balance sheet information so that we can construct more accurate portfolio weights. The SCF’s main drawback is that it consists of repeated cross-sections instead of following individuals over time.

Throughout this paper, the sample includes only households aged 25 to 84. To define household age, education, and gender, I use the household head. In the PSID households from the Survey of Economic Opportunity and Latino Samples are excluded to obtain a representative sample of households. As both the PSID and the SCF are well-known data sets, I omit a detailed description of them, but refer to Gianetti and Wang (2016) for a discussion of measuring participation in the PSID and Athreya et al. (2017) for similar introduction to the SCF.

2.1 Estimation of Life-Cycle Profiles

When calibrating and estimating life-cycle models, one must take a stand on whether one should control for time effects or cohort effects since age, birth year, and calendar year are perfectly collinear (Ameriks and Zeldes, 2004). There is no consensus in the literature on which approach to use. The resulting patterns are, in general, largely similar but with larger deviations for older households. Since house prices and stock
returns have large year-to-year fluctuations with similar effects across age, I control for time effects and assume that cohort effects are zero.

To construct the age-profiles, I bin age into five year intervals from [25, 29], [30, 34], ..., [80, 84] to reduce small-sample noise. I use household weights normalized within years to equalize the weight on each year. I drop households that belong to the top 1% and bottom 1% of the wealth distribution within each age group. For all life-cycle statistics in this paper, I use weighted linear regression with a full set of age and year fixed effects. I then find the predicted value for each age and year and find the average predicted value across all years in the sample.

2.2 Life-Cycle Participation, Ownership and Exit

I now turn to study how the participation and homeownership rates evolve over the life cycle in the PSID and the SCF. The left panel of Figure 1 uses the PSID and reveals limited stock market participation at all ages. It starts at 10% among the youngest households, increases gradually to 55% at age 65 before it decreases slightly during retirement. Moreover, homeownership increases quickly from 15% among the young to 70% for households aged 45, with a peak at 80% at age 67 before households start to liquidate housing in retirement. The right panel plots the same data from the SCF. Broadly speaking, the patterns are the same, although the participation rate in the SCF is higher for younger households.

2.3 Entry and Exit Over the Life-Cycle

To better understand what drives the low participation rate in stock markets, I now turn to participation dynamics among US households using data from the PSID (we cannot observe dynamics in the SCF since it is a cross-section). Figure 2, left panel, plots the entry and exit rates from stocks over the life cycle, controlling for time.

As is well known, this class of models cannot easily match the upper tail of the wealth distribution.
effects. We can see that the exit rate is 35% at age 25, and then gradually decreases towards retirement age, at which point the exit rate starts to increase. Similarly, we see that the entry rate largely displays the opposite pattern, starting at 8% at age 25, gradually increasing until retirement age, and then weakly decreasing. The second panel plots the entry and exit into homeownership. We can see that middle-aged owners rarely become renters (exit) and that the entry rate in homeownership is relatively stable at 10-15% before retirement. Once households retire, we see that the entry rate to ownership drops while the exit rate increases. The largest difference in dynamics between homeownership and stock market participation is the high exit rate among stock owners.

2.4 Regressions: Determinants of Stock Market Participation

Which variables predict stock market participation? Participation is related to homeownership using the following linear empirical model:

\[ Participation_{it} = \beta \times Homeowner_{it} + \gamma X_{it} + \varepsilon_{it}, \]  

(1)
These figures plots the exit and entry rates in stocks and homeownership. The exit rate is defined as the fraction of households who own stocks (primary residence) in year $t$ and do not in $t + 2$, and are observed in both periods, and vice versa for the exit rates.

where participation is measured as holding any amount of stocks in liquid accounts or IRA’s or the alternative measure which excludes stock in IRA’s. This specification mirrors the one used in Gianetti and Wang (2016), and who also use the PSID. The main set of controls include logged income and logged wealth, marriage status, education, lagged participation, logged age, and logged family size, in accordance with the existing literature (e.g., Guiso, Sapienza, and Zingales (2008); Gianetti and Wang (2016)). In some specifications, I include individual-level fixed effects. Some variables are logged for ease of interpretation and to lessen the impact of outliers. The results are reported in Table 1. As Gianetti and Wang (2016) I use a linear probability model due to a large number of fixed effects and for ease of interpretation.

The first two columns regress participation (in any account), while columns 3-4 ignores participation in retirement accounts. Columns 2 and 4 include household fixed effects to control for unobserved heterogeneity, such as risk aversion. From column 2, we see that being a homeowner is associated with a 5.6 percentage point decrease in participation probability, indicating a strong substitution between stocks and housing. The effect is somewhat lower when ignoring stocks in retirement accounts. Other coefficients have the expected signs, with income and wealth increasing participation:
<table>
<thead>
<tr>
<th></th>
<th>(1) Participation</th>
<th>(2) Participation</th>
<th>(3) Participation (w/o IRA)</th>
<th>(4) Participation (w/o IRA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeowner=1</td>
<td>-0.057***</td>
<td>-0.056***</td>
<td>-0.044***</td>
<td>-0.023***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.003)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>-0.028***</td>
<td>0.171***</td>
<td>-0.010**</td>
<td>0.132**</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.043)</td>
<td>(0.004)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Married=1</td>
<td>0.027***</td>
<td>0.041***</td>
<td>0.009**</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.008)</td>
<td>(0.003)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Log(Family Size)</td>
<td>-0.044***</td>
<td>-0.013**</td>
<td>-0.025***</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.005)</td>
<td>(0.002)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Log(Income)</td>
<td>0.033***</td>
<td>0.013***</td>
<td>0.014***</td>
<td>0.008***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Log(Wealth)</td>
<td>0.055***</td>
<td>0.050***</td>
<td>0.032***</td>
<td>0.023***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>College=1</td>
<td>0.108***</td>
<td></td>
<td>0.078***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
<tr>
<td>Lagged Part.</td>
<td>0.432***</td>
<td>0.023***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged Part.</td>
<td></td>
<td></td>
<td>0.479***</td>
<td>-0.001</td>
</tr>
<tr>
<td>(w/o IRA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>74984</td>
<td>74984</td>
<td>58563</td>
<td>58563</td>
</tr>
<tr>
<td>Individual FE</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>Year and State FE</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Within ( R^2 )</td>
<td>0.020</td>
<td>0.055</td>
<td>0.003</td>
<td>0.025</td>
</tr>
</tbody>
</table>

This table presents the relationship between household characteristics and stock market participation. Standard Errors in parentheses. Clustered at the family level. * \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \).
a 10% increase in wealth increases the probability of participation by 5.0 percentage points when controlling for fixed effects. Controlling for fixed effects has minor impacts on the other control variables, except for the effect of lagged participation. This suggests that participation in one period does not lead to participation in later periods by itself but that it is a combination of individual-specific characteristics and other financial observable variables that drive a household’s participation decision.

2.5 Regressions: Determinants of Stock Market Exit

Why is the exit rate so high? I report results from four different regressions, reported in Table 2. The empirical model is the same as in equation (1) but with stock market exit as the dependent variable. The sample is thus limited to households who participated in \( t - 2 \). We see that after controlling for other variables, homeowners are 4.3 percentage points more likely to exit stock markets. The impact of wealth is higher on exit than on participation: A 10% decrease in wealth increases the probability of exit by 8.3 percentage points, while it decreases the probability of participation by ‘only’ 5.5 percentage points. Marriage, family size, and income are only weakly related to the exit rate. Why is homeownership associated with stock market exit? Since the two first specifications include individual fixed effects, the coefficient on homeownership must be interpreted with care. The estimate reflects the effect of homeownership only among households for whom we observe house tenure changes (renter → owner and owner → renter).

To decompose the effect of homeownership, I limit the sample to households who were participating and renting in \( t - 2 \). The results are reported in specifications (3) and (4) of Table 2. I use the same control variables but now measure the relationship between exit and becoming a homeowner. New homeowners have a 12-16 percentage points higher exit rate, relative to a mean exit rate of 21%. The relationship is higher when considering exit from non-retirement accounts, possibly as a result of
Table 2: The Determinants of Stock Market Exit

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Renters in ( t - 2 )</th>
<th>Renters in ( t - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Exit</td>
<td>(2) Exit (w/o IRA)</td>
<td>(3) Exit (w/o IRA)</td>
<td>(4) Exit (w/o IRA)</td>
</tr>
<tr>
<td>Homeowner=1</td>
<td>0.043**</td>
<td>0.030</td>
<td>(0.015)</td>
</tr>
<tr>
<td>New Homeowner=1</td>
<td>0.124***</td>
<td>0.160***</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Employed=1</td>
<td>0.029**</td>
<td>0.038*</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Married=1</td>
<td>-0.047*</td>
<td>-0.009</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Log(Age)</td>
<td>0.698***</td>
<td>0.986***</td>
<td>(0.130)</td>
</tr>
<tr>
<td>Log(Family Size)</td>
<td>0.028*</td>
<td>0.023</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Log(Income)</td>
<td>-0.017**</td>
<td>-0.013</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Log(Wealth)</td>
<td>-0.083***</td>
<td>-0.077***</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Observations</td>
<td>20103</td>
<td>10485</td>
<td>3559</td>
</tr>
<tr>
<td>Within ( R^2 )</td>
<td>0.076</td>
<td>0.089</td>
<td>0.145</td>
</tr>
</tbody>
</table>

This table presents the relationship between household characteristics and exit from stock markets in a linear probability model. To be in the sample households must have owned stocks in year \( t - 2 \). Standard errors in parentheses. Clustered at the family level. * \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \).

The effect of wealth is the same as when using the full participant sample. The effect of age, marriage status, family size, and income are all statistically non-significant.

Overall, the evidence in this section has three main implications. First, it suggests that the high exit rate partially explains the low participation rate. Second, their more liquid nature.\(^4\) The effect of wealth is the same as when using the full participant sample. The effect of age, marriage status, family size, and income are all statistically non-significant.

\(^4\)While retirement accounts are less liquid, they often have rules allowing a withdrawal in times of hardship or special circumstances. For example, first-time homeowners can withdraw penalty-free from an IRA to cover a down payment.
I find that homeownership is a substitute for stock market participation. Third, homeownership is associated with stock market exit once other variables are held constant. These three facts taken combined suggest that a model with an endogenous house tenure decision combined with an endogenous entry/exit in stock markets may account for low stock market participation rates over the life cycle.

3 A Model of Participation and Homeownership

The previous sections establish novel facts about the life-cycle exit and participation decisions of US households. Existing models can account for the hump-shaped participation rate, but not for the joint patterns of homeownership, stock market participation, and entry/exit in stock markets. In this section, I present a life-cycle model that explains these patterns. Before laying out the environment, technology, and preferences, I first provide a birds-eye of the model and describe the main two mechanisms through which housing reduces participation and increases exit.

To facilitate comparisons with the literature, I build on Fagereng et al. (2017), which in turn extends the workhorse model of Cocco et al. (2005). Relative to Fagereng et al. (2017), I add a housing market where households can choose to rent or own. The no-borrowing condition is also relaxed to allow for collateralized borrowing (mortgages). As in Fagereng et al. (2017), tail events and per-period participation account for the decreasing participation rate and portfolio weights among older households. Two main mechanisms improve the model’s predictions.

First, housing decreases participation among younger and middle-aged households by tying up wealth in an illiquid asset. This effect is strengthened by the illiquidity of housing (through transaction costs): If a household recently bought a house, they have little liquid wealth. If they receive a bad income shock or a bad stock return shock, they may not be able to repay the mortgage payment in the next period and must downsize. This will also help generate more exit: Some wealthy
households who own housing will receive bad income or stock return shocks, so that wealth drifts down. At some point, they become liquidity constrained and so exit the stock market. Without housing, this only happens if households drift towards zero wealth, a very rare event both empirically and in the model.

Second, As noted in Vestman (2019), renters will, all else equal, have more to gain by participation in stock markets. However, tail events reduce the renters’ willingness to participate. Intuitively they face the trade-off of investing in stock markets to gain the equity premium to afford the down payment a little sooner. However, if a tail event materializes the dream of homeownership will be delayed when the intended downpayment was substantially invested in stocks.

3.1 Households and Utility

Households enter the economy at age 25 and retire at age $T^r$. The probability of death between age $a$ and $a + 1$ is $\pi_a$, but households die with certainty at age 100. Households have time-separable preferences with discount factor $\beta$ over consumption $c_a$ and housing $h_a$, and the utility function is assumed to have a constant relative risk aversion form with a Cobb-Douglas aggregator:

$$U(c_a, h_a, o) = \left(\frac{\eta^\gamma \chi_o h_a^{(1-\eta)}}{1-\gamma}\right)^{1-\gamma},$$

where the utility function is allowed to depend on house tenure through the parameter $\chi_o$ that equals unity if the household is renting and $\chi_1$ if the household owns. The parameter $\gamma$ measures risk aversion and $\eta$ relates to the budget share of consumption expenditure. Households who do not survive between age $a$ and $a + 1$ derive warm glow utility from accidental bequest. A household that dies with $x_{a+1}$ in networth receives $v(x_{a+1})$ utils, where

$$v(x_{1+1}) = \psi \left(\frac{\max\{x_{a+1}, 0\}}{1-\gamma}\right)^{1-\gamma}.$$
The max operator is introduced for two reasons. First, in the US, households can die with negative net debt, but that debt is not passed on to heirs. Second, with housing risk as introduced later, households can end up underwater on the mortgage (market value less than their debt), and with negative net worth. However, the CRRA utility function is defined only for strictly positive numbers.\textsuperscript{5}

3.2 Market Structure and Stochastic Processes

3.2.1 Financial Markets

There are two financial assets, a risk-free one-year bond $b_a$ and risky stocks $s_a$. The bond earns a constant interest rate $r_f$ if positive, but debt (mortgage) has a mortgage premium of $r_m$. Borrowing is only available to those who own or buy owner-occupied housing, and the maximum mortgage is limited by the downpayment requirement $d$

\[ b_{a+1} \geq -(1 - d)p_a h_{a+1}, \quad (4) \]

where $p_a h_{a+1}$ denotes the market value of the house.

The risky asset has uncertain returns, defined as the sum of the risk-free rate, the risk premium $r_p$ and a financial shock $\epsilon_s^t$. The shock follows a normal distribution augmented with a small probability $p_{\text{tail}}$ of stock market crashes:

\[ r_t = r_f^t + r_p^t + \epsilon_s^t, \quad \text{with} \quad \epsilon_s^t \sim \begin{cases} r_{\text{tail}} & \text{with probability } p_{\text{tail}}, \\ \mathcal{N}(0, \sigma_s^2) & \text{with probability } 1 - p_{\text{tail}}. \end{cases} \quad (5) \]

Households who participate in stock markets ($s_a > 0$) must pay per-period participation cost $q$.

\textsuperscript{5}I actually therefore use $\max\{x_{a+1}, \epsilon\}$ where $\epsilon$ is a small number when solving the model numerically.
3.2.2 House Market

In the model house prices are linear in house quality $h$, and follow a stochastic process with drift $\mu$ and housing market uncertainty $\varepsilon^h$, as well as stock market correlation $\theta^s_h\varepsilon^s$:

$$R^h_{t+1} \equiv \frac{p_{t+1}}{p_t} = \exp(\mu + \varepsilon^h_{t+1} + \theta^h_{s}\varepsilon^s_{t+1}), \quad \sigma_t \sim \mathcal{N}(0, \sigma^2_h),$$

(6)

where $p_a$ is the house price per square meter. The parameter $\theta^h_s$ pins down the correlation between the stock market and housing market. Further, the rental price is assumed to be a constant fraction $f$ of the market value. To ease computations the rental market consists of a single unit of quality $h_{rent}$, and the owner-occupied markets has two sizes, $h_{small}$, $h_{large}$. House transactions are settled immediately, so a household that enters a period as a renter can immediately become an owner, and vice-versa.

Buying owner-occupied housing entails a purchase and moving cost $m$ that is proportional to the market value of the house.

3.2.3 Income

Income $w_{i,a}$ of individuals $i$ at age $a$ consists of one idiosyncratic transitory shock $\varepsilon_i$, an idiosyncratic persistent shock $v_i$, a deterministic life-cycle component $f_a$ as well as the aggregate shocks:

$$\log(w_{i,a}) = f_a + v_{i,a} + \varepsilon_{i,a} + \theta_{w^h}^h\varepsilon_{i,a}^h + \theta_{w^s}^s\varepsilon_{i,a}^s, \quad a < T^R, \varepsilon_{i,a} \sim \mathcal{N}(0, \sigma^2_\varepsilon),$$

(7)

where the $\theta$'s pin down the correlation between income and house prices and stock returns. The persistent component $v_{i,a}$ follows a first-order autoregressive (AR(1)) process with normal innovations:

$$v_{i,a} = v_{i,a-1} + \nu_{i,a}, \quad \nu_{i,a} \sim \mathcal{N}(0, \sigma^2_\nu).$$

(8)
Retired households face no labor income risk. To capture social security and pension benefits retired households receive a fraction $\phi$ of their wage in the last period of working life:

$$w_{i,a} = \phi w_{i,T^r} \forall a \geq T^r.$$  

Going forward, the subscript $i$ is omitted unless necessary.

### 3.2.4 Welfare System

Since house prices are growing at rate $\mu$ and subject to log-normal shocks, the house price is unbounded above. At the same time income is stationary, and so for very high price realizations some households may unable to afford a place to live. I therefore assume that the government runs a welfare system that guarantees that the minimum after-tax income $w$ is 5% larger than the market price of rental housing. If no such assumption is placed, the model must include either a reflecting barrier for house prices or explicitly model homelessness.

### 3.3 Borrowing Constraints and Law of Motion

The model is best understood from the household’s budget constraints and the law of motion for wealth. Households have two sources of wealth: Financial wealth from stocks or bonds and owner-occupied real estate. Let $x_a$ denote a household’s wealth at age $a$.

#### 3.3.1 Evolution of Cash-on-Hand

If the household chose to rent at the beginning of the period, the realized net worth in the next period is given by realized gains on bonds and stocks:

$$x_{a+1} = s_{a+1}(1 + r_{a+1}) + b_{a+1}(1 + r^f). \tag{9}$$

Homeowners’ future wealth depends additionally on the next period market value
of the house, net of depreciation:

\[ x_{a+1} = s_{a+1}(1 + r_{a+1}) + b_{a+1}(1 + r^f) + p_{a+1}h_{a+1}(1 - \delta). \tag{10} \]

### 3.3.2 Budget Constraints

In each period, the households choose how to spend their disposable wealth \( x_a + w_a \) on consumption \( c \), bonds \( b \), stocks \( s \) and housing services \( h \). If a household decides to rent it pays the rental price \( f ph \), as well as adjustment costs on housing

\[ c_a + (s_{a+1} + 1_s q) + b_{a+1} + fp_a h_a + adj(h_a, h_{a+1}) = x_a + w_a \tag{11} \]

where we use \( 1_s \) to denote an indicator functions that equals one households participate in stock markets (\( 1_s \equiv 1_{s_{a+1} > 0} \)).

Households that choose to own a house in the current period pay the market price of the house, as well as adjustment costs

\[ c_a + 1(s_{a+1} + q) + b_{a+1} + p_a h_a + adj(h_a, h_{a+1}) = x_a + w_a. \tag{12} \]

Since house prices are stochastic, we see that house price risk manifests differently for renters and owners. Renters face expenditure uncertainty while owners face wealth uncertainty.

### 3.4 Recursive Formulation

All households have several choices: consumption \( c \), portfolio choices between bonds \( b \), stocks \( s \), and housing \( h \), as well as the discrete ownership and participation decisions. Due to the discrete choices, the decision problems separates into two parts: First, households choose whether to rent, buy a house or if they already own to stay. Second, households choose whether to participate in the stock market. Conditional
on these discrete choices, they choose optimal consumption, savings, and portfolio weights. The discrete nature of the problem lends itself to solving the decision problem in parts, conditional on choosing to buy \( B \), rent \( R \), or staying \( S \) in the house.

Since house transactions are immediate a homeowner who sells his house with wealth \( x_a \) is identical to a renter with wealth \( x_a \). Denote a renter’s value function by \( V^R_a(x_a, v_a, p_a) \), the value function of households who buys a new house \( V^B_a(x_a, v_a, p_a) \), and the value function of owners who stay by \( V^S_a(x_a, v_a, p_a, h_a) \). For the rest of the paper I omit the age subscript on variables, and use primes to denote the \( a + 1 \) subscript. A household of that owns at age \( a \) then chooses optimally between these tree alternatives:

\[
V_a(x, v, p, h) = \max_{R,B,S} \{ V^R_a, V^B_a, V^S_a \}. \tag{13}
\]

A renter faces only the choices between renting and buying. It is worth mentioning that if one omits housing, for example, by setting \( h = 0 \), that the model collapses exactly to the one in Fagereng et al. (2017), and thus also nests Cocco et al. (2005). See appendix B.2 for more.

### 3.4.1 Stayers’ Decision Problem

I now present the decision problem of a stayer. The decision problem of renters and buyers are in appendix B.1. A household who enters the period with wealth \( x \), has persistent human capital \( v \), observes house prices \( p \) and owns a house of size \( h \) chooses consumption \( c \), levels of stock investment \( s' \) and the net bond position \( b' \). If
he holds a positive amounts of stocks he pays the participation cost $q$:

$$V_a^S(x, v, p, h) = \max_{c, b', s'} \{u(c, h) + \beta \mathbb{E}_a[\pi_a V_{a+1}(x', v', p', h) + (1 - \pi_a)(x')]\}$$

$$x + w = c + 1(s' + q) + b' + ph$$

$$x' = s'(1 + r') + b'(1 + r') + hp'(1 - \delta)$$

$$\{c, s'\} \in [0, \infty)^2, b' \geq -(1 - d)ph$$

(14)

3.5 Simulation

To take the model to the data, I simulate a large panel of households using the decision rules from the model outlined above. I now discuss how the simulated panel is constructed.

3.5.1 Initial Conditions

When simulating households, we need to simulate households' initial wealth $x$, human capital $v$, price level $p$, and homeownership. I draw human capital from the stationary distribution implied by the process in equation (8). I assume that the initial house price can take three values. I find the benchmark price by excluding house values below the first and above the top percentiles or with 20 or more rooms. I then run a regression of house values on the number of rooms with year fixed-effects and without a constant. The estimated slope is $36,000$, which I use as the initial benchmark price per room. Next, I run the same regression on a sample of ‘coastal-urban states’ (CA, NY, WA, MA) and another on the central Mid-West (WI, IO, MN, MI, IN, OH), which yields prices of $49,000$ and $29,000$ respectively. 70% of the economies use the initial national price while the remaining 30% are equally divided into the other two initial prices. For initial wealth and homeownership, I draw non-parametrically from the data. Let $v(x, h)$ denote the discretized PDF of net-worth and homeownership. All households who are owners are assumed to own a small house, and net worth is
censored below by $-$15,000. I then group households who are 25 into 20 quantiles of wealth and whether they own or rent. The distribution is plotted in Figure 6b. Within each wealth bin, I use the mean wealth as a value for households drawn to be in that bin.

3.5.2 ‘Aggregate’ Risk

There are three sources of risk in the model: idiosyncratic income risk, aggregate stock returns, and aggregate house price fluctuations. To understand each channel’s relative importance, I follow the simulation procedure in Dahlquist, Setty, and Vestman (2018). First, I simulate the income shock realizations for 1250 individuals \( \{v_{i,a}, \varepsilon_{ia}\}_{a=25}^{T_r} \). I then simulate 16 realizations of stock returns \( \{r_{j,a}\}_{a=25}^{100} \) and 40 house prices sequences \( \{p_{k,a}\}_{a=25}^{100} \), which generates 640 different ‘economies’ (\( j \times k \)). I then simulate the behavior of these 1250 individuals in the 640 economies for a simulated panel with 800,000 individual \( \times \) economy realizations. As the simulated outcomes are sensitive to aggregate outcomes, it is important to simulate multiple economies to avoid spurious results.

4 Estimation

The model is estimated in two steps. In the first step I estimate prices and stochastic processes directly from the data or set them to the standard parameter values in the literature. In the second step I estimate the preference parameters, probability of stock market tail events and the participation cost by simulated method of moments (SMM).

4.1 Externally Calibrated Parameter Values

Table 3 lists all exogenously calibrated parameters along with their sources.
Table 3: Model Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial markets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk free rate</td>
<td>$r_f$</td>
<td>0.02</td>
</tr>
<tr>
<td>Equity premium</td>
<td>$r_p$</td>
<td>0.04</td>
</tr>
<tr>
<td>Stock return std. deviation</td>
<td>$\sigma_s$</td>
<td>0.157</td>
</tr>
<tr>
<td>Tail event return</td>
<td>$r_{tail}$</td>
<td>-0.5</td>
</tr>
<tr>
<td>Mortgage premium</td>
<td>$r_m$</td>
<td>0.0169</td>
</tr>
<tr>
<td>Correlation income &amp; stocks</td>
<td>$\theta_{sw}$</td>
<td>0.0</td>
</tr>
<tr>
<td>Correlation income &amp; prices</td>
<td>$\theta_{sw}$</td>
<td>0.0</td>
</tr>
<tr>
<td>Correlation stocks &amp; prices</td>
<td>$\theta_{sh}$</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>Labor Market</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto-correlation</td>
<td>$\rho$</td>
<td>0.95</td>
</tr>
<tr>
<td>Transitory shocks std. dev.</td>
<td>$\sigma^2_{\xi}$</td>
<td>0.08</td>
</tr>
<tr>
<td>Persistent shocks std. dev.</td>
<td>$\sigma^2_{\nu}$</td>
<td>0.018</td>
</tr>
<tr>
<td>Start of retirement</td>
<td>$T^R$</td>
<td>69</td>
</tr>
<tr>
<td>Replacement ratio</td>
<td>$\hat{\phi}_{ret}$</td>
<td>0.758</td>
</tr>
<tr>
<td>Deterministic wage profile</td>
<td>$f_a$</td>
<td>fig. 6a</td>
</tr>
<tr>
<td><strong>Housing Market</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>House price drift</td>
<td>$\mu$</td>
<td>0.015</td>
</tr>
<tr>
<td>House price std. deviation</td>
<td>$\sigma_h$</td>
<td>0.093</td>
</tr>
<tr>
<td>House price depreciation</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>Minimum down payment</td>
<td>$d$</td>
<td>0.15</td>
</tr>
<tr>
<td>Transaction cost</td>
<td>$mc$</td>
<td>0.08</td>
</tr>
<tr>
<td>Rent-to-house value</td>
<td>$f$</td>
<td>0.05</td>
</tr>
<tr>
<td>Rental size</td>
<td>$h_{rent}$</td>
<td>4</td>
</tr>
<tr>
<td>Owner-occupied sizes</td>
<td>$h_{small}, h_{large}$</td>
<td>(5,8)</td>
</tr>
<tr>
<td><strong>Simulation &amp; Other</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Survival rates</td>
<td>$\pi_a$</td>
<td>fn.</td>
</tr>
<tr>
<td>Wealth-Ownership joint distr.</td>
<td>n/a</td>
<td>fig 6b</td>
</tr>
</tbody>
</table>

*Note:* The table lists the parameter values for the correlation parameter. These parameters gives a correlation of 0.0, 0.22 and 0.0 between stock market and labor market shocks, stock market and housing shocks, and housing and labor market shocks.
4.1.1 Return Processes

I take the parameters that govern the stock market returns from Cocco et al. (2005), and so the risk-free rate $r^f = 0.02$, the equity premium $r^p = 0.04$, and the standard deviation of stocks $\sigma_s = 0.157$. To find the mortgage premium I calculate the average spread between the average interest rate on 30-year fixed rate mortgages and 10-year treasury bills between 1971 and 2018, which yields $r^m = 0.0169$. Finally, I set the return in case of tail events to be $r_{tail} = -0.5$.\(^6\) The probability of tail events is estimated jointly with the preference parameters.

4.1.2 Correlations of shocks

To calibrate the correlation between house prices, stock returns and wage shocks I use historical data: the seasonally adjusted monthly S&P/Case-Shiller House Price Index, monthly Wilshire 5000 Total Market Full Cap Index and yearly median weekly real earnings, respectively. To create yearly indexes I take the average value within each year. I then find the year-on-year percentage change of each index and calculate the correlation. This yields correlations of 0.017 between house prices and wages ($p = 0.93, N = 31$), 0.215 between house prices and the stock market ($p = 0.24, N = 31$) and 0.003 between wages and stocks ($p = 0.81, N = 38$). Since all correlations are low and not statistically significant I set $\theta_h^s = \theta_w^h = \theta_w^s = 0$.

4.1.3 Income Process

To find the retirement age $T^r$ I count disabled households as retired, but omit households who retire before they turn 51. I then find that the average age at which households became permanently retired is 69, that is where no subsequent spells of labor market participation is observed. To find the retirement income shifter $\phi_{ret}$ I take the average income of a household in the two preceding waves before retirement.

---

\(^6\)Fagereng et al. (2017) estimate $p_{tail} = -0.485$ in Norway, but it is not clear how one estimates the typical return of rare events.
and divide by the average income in the three succeeding waves. The average ratio is 75.8% omitting the 1st and 99th percentiles.

Next, I find the common deterministic life-cycle component of income $f_a$ by taking the sample of households aged 25-67 who are employed, looking for jobs, temporarily sick, or disabled and run OLS on log earnings with a full set of age and year dummies. I then find the average log earnings at each age equally weighting the years. I then fit a fifth-order polynomial to the resulting (non-logged) predicted earnings. The results are reported in Figure 6a.

I take the parameters governing the stochastic component of labor income from the literature. Many papers (e.g. Cocco et al. (2005), Cooper and Zhu (2016)) estimate and solve the model for different education groups. However, in this paper I solve the model for the general population, ignoring heterogeneity in education. This is done for two reasons. First, for the purposes of this paper it is necessary to observe both stock market entry and exit, as well as transitions in homeownership. These are infrequent events, and so the sample is too small if limiting the sample to only use on education group. Second, the introduction of housing increases the computational burden significantly, and omitting heterogeneity in education keeps the problem tractable. I therefore set the persistence $\rho = 0.95$, variance of persistent shocks $\sigma^2_\nu = 0.018$ and the variance of the transitory shock $\sigma^2_\varepsilon = 0.08$. These parameter values lies squarely in the range considered in the literature (Carroll and Samwick, 1997; Cocco et al., 2005; Cooper and Zhu, 2016; De Nardi, Fella, and Paz-Pardo, 2019)

### 4.1.4 House Prices

I base the parameters governing the house price growth from Cocco (2005). He sets average price growth to be 1% and its standard deviation to be 0.062. To capture the development of house prices since 2000 I increase both the drift and standard deviation by 50%, to $\mu = 0.015, \sigma_h = 0.093$. I take depreciation $\delta = 0.025$ as estimated
in Harding et al. (2007). Finally, I take the rental cost $f$ to be 5% (Davis et al., 2008). Further, the median rental unit has 4 rooms. Finally I set the transaction cost $mc$ and minimum downpayment $d$ to be 8% and 15%, respectively, as in Cocco (2005). In section E.1, I study the impact of different minimum downpayment requirements. The PSID records the number of rooms (excluding bathrooms) in both owner-occupied and rental housing. The 25th percentile of owner-occupied housing has 5 rooms while the 75th percentile has 8.

4.1.5 Remaining Parameters

I take the conditional survival probabilities from the 2004 Social Security Administration Life Table. For the simulations a crucial component is the initial distribution of wealth, ownership and productivity. I set the initial distribution of productivity $v$ to be the stationary distribution of $v$. I estimate the joint distribution of initial wealth $x_{25}$ and homeownership $h_{25}$ non-parametrically from the data.

4.2 Internal Estimation

I perform a structural estimation of the preference parameters of the model: risk aversion $\gamma$, discount factor $\beta$, participation cost $q$, housing expenditure share $\eta$, ownership preference $\chi_1$, bequest parameters $\psi_0, \psi_1$ and the subjective probability of tail events $p_{\text{tail}}$ by the Simulated Method of Moments (SMM). Denote the parameter vector by $\omega \in \mathbb{R}^8$. Given a candidate parameter vector $\omega$, I solve the model and obtain simulated moments $m(\omega)$ and compare them to the empirical moments $m$. I search for a vector $\hat{\omega}$ that minimizes the weighted deviation:

$$\hat{\omega} = \arg \min_\omega \{[m(\omega) - m]'W[m(\omega) - m]\}, \quad (15)$$

where $W$ denotes the weight matrix, chosen to be $1/m$ on the diagonal and zeros elsewhere, to normalize each moment. Details on the estimation procedure are in
appendix C.3. The estimation targets the participation rate, homeownership rate, conditional portfolio weight on stocks and net worth over the life cycle, for households aged 25-79. A discussion of the estimation of the life cycle profiles is in appendix 2.1. I use the SCF to estimate the life cycle profile of portfolio weights among participants since the PSID does not include information on stock holdings within IRAs, see section 2 for more. The estimation is over-identified, with 6 parameters and 44 moments: four life-cycle moments in 11 age-bins ([25,29],[30,34],...,[75,79]).

4.2.1 Estimation Results

Table 4 reports the estimated parameters and the most informative moments for each parameter, while Figure 3 plots the simulated and true data moments ($m(\hat{\omega})$ and $m$). The plots reveal that the model largely fits the data quantitatively. We
can see that the model fits the participation rate well. The model underpredicts portfolio weights for households over the age of 45 due to the large estimated risk aversion and high probability of tail events. The model matches wealth accumulation up to age 55 but generates a too large decline in wealth after retirement. The model abstracts from several aspects of retirement and models it as an exogenous change at age $T^R$. Finally, the model qualitatively matches the homeownership rate but overpredicts among middle-aged households. The parameter values for risk aversion and discount factor ensure that virtually all households save substantial amounts to fund consumption in retirement, and so most households cross the endogenous rent/own wealth threshold.

Table 4: Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Housing</th>
<th>No Housing</th>
<th>Identifying Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$ Participation cost</td>
<td>0.124</td>
<td>0.441</td>
<td>Participation</td>
</tr>
<tr>
<td>$\beta$ Discount Factor</td>
<td>0.937</td>
<td>0.907</td>
<td>Net Worth, Participation</td>
</tr>
<tr>
<td>$\gamma$ Risk Aversion</td>
<td>4.518</td>
<td>4.111</td>
<td>Portfolio W., Net Worth</td>
</tr>
<tr>
<td>$p_{tail}$ Tail Events</td>
<td>0.041</td>
<td>0.044</td>
<td>Portfolio W., Participation</td>
</tr>
<tr>
<td>$\chi_1$ Owner Preference</td>
<td>1.16</td>
<td>–</td>
<td>Homeownership</td>
</tr>
<tr>
<td>$\psi$ Bequest Motive</td>
<td>706.57</td>
<td>171.74</td>
<td>Net Worth (old households)</td>
</tr>
</tbody>
</table>

Identifying moments are listed in approximate order of importance.

The model struggles to explain behavior among middle-aged households. Since stock market participation is largely driven by wealth, and the participation cost is constant across the life-cycle, the model must trade of increasing participation among the young by increasing $\beta$ and decreasing the participation cost $q$ or decrease participation among the middle-aged by decreasing $\beta$ or increasing $q$. The only other mechanism is housing: since ‘house-poor’ homeowners are unlikely to participate in the stock market, the estimation increases the homeownership rate among the middle-aged to decrease their participation. The result is that more middle-aged
households have low home equity and prioritize building more home equity before participating in the stock market.

Finally, Table 4 lists the point estimates and the main identifying moments. The third column also reports the parameter estimates from a model without housing (see section 4.3). The participation cost is estimated to be 0.124, which can be interpreted as $124 in 1995 dollars. The model estimates a discount factor of 0.937, a risk aversion of 4.52, a probability of tail events of 4.1%, which all are lower than the estimates in Fagereng et al. (2017) but comparable to those in Catherine (2019). The ownership preference premium is quite low 1.16 relative to other life-cycle papers (e.g., Corbae and Quintin, 2015; Chang, 2020). Two model mechanisms drive down the estimated preference shifter: First, the largest rental unit is the small house size, so wealthy houses who want to equate the marginal benefits and costs of consumption and housing must be homeowners once they are sufficiently wealthy. Second, with stochastic prices and constant rental price ratios, renting is risky, and becoming a homeowner provides insurance against expenditure shocks.

4.3 Model Improvement with Housing

While the estimated model does well at matching the observed life-cycle outcomes, it is instructive to see what happens when housing is omitted. I redefine net worth to financial wealth to keep the model consistent with the data. The portfolio weight is similarly redefined to stocks over financial wealth instead of over net worth. In addition, I remove the minimum income floor as in Fagereng et al. (2017). Since this model omits housing, the parameters $\chi_1$ and $\eta$ are redundant. The estimation thus mirrors the estimation in Fagereng et al. (2017) in all aspects, except that the estimation targets US data. Figure 4 plots the estimated life-cycle patterns, and Table 4 lists the estimated coefficients.

---

7When estimating the life-cycle moments, I follow the procedure described in 2.1, but I recoded financial wealth to missing for households in the top/bottom 1% of financial wealth, before dropping households in the top/bottom 1%.
Figure 4: Estimation Without Housing - Targeted Life Cycle Moments

These figures plots the targeted life-cycle moments in the simulated data in the model with and without housing, as well as the empirical moments.
Comparing the estimated outcomes, it is clear that the model with housing matches the participation rate significantly better, even though it targets an additional moment. The peak participation rate is too high without housing, while the participation rates for young and old households are too low. The model without housing is able to largely match net worth, but still with too strong withdrawal after retirement. Both models are largely able to match the data for middle-aged households, but the largest improvement for the model without housing is among younger households. This is intuitive: The transition from renting to owning happens for young and middle-aged households. Further, the model and results in Fagereng et al. (2017) show that the model without housing can qualitatively match the dual adjustment of old households (exit and increased portfolio weights among participants). The housing market in this paper omits several important factors for older and retired households, such as reverse mortgages and increased depreciation, which could improve the model’s ability to match data among older households.

Finally, from Table 4, we see that the participation cost increases to 0.441 from 0.124 while the discount factor decreases to 0.907 from 0.937. At the same time, risk aversion falls slightly to 4.111, and the probability of tail events increases to 4.4% from 4.1%. The main takeaways are: First, a model with housing can quantitatively match the participation rate over the life cycle with a low participation cost ($124), relative to a model without housing. Second, the sum of the squared deviation between the model and empirical participation rates decreases by 71% from 60.5 to 17.62 when housing is included. Third, the model fit, as calculated by the sum of squared percentage deviations, decreases by 40% by including housing, even though it only introduces one new estimated parameter for 11 new moments (homeownership over the life-cycle).
5 Non-Targeted Moments

I now turn to discuss the models’ performance on matching non-targeted moments to highlight the robustness of the model.

5.1 Stock Market Entry and Exit

This paper proposes a new channel (entry and exit) to explain limited stock market participation. Figure 5 plots the simulated and true two-year entry/exit rates over the life-cycle. The model with housing does remarkably well at matching the qualitative and quantitative patterns with hump-shaped entry and u-shaped exit. In the model, almost all renters who become owners sell their stocks, leading to a too high simulated exit rate initially. However, without housing, we see that the model generates too little exit in middle age. The only reason households exit without housing is that their wealth declines below the participation threshold due to a sequence of bad income or stock returns. However, at middle-age, the threshold is low, and most households have accumulated significant savings, and there is virtually no exit without housing. I study the role of the minimum downpayment level in Appendix E.1. I show that entry is increasing in downpayments (i.e., the highest entry rate is obtained when households must pay for the house without borrowing) and that exit is decreasing.

5.2 Micro-Level Behavior

The model matches aggregate behavior well. I now use the simulated panel to estimate the causal effects of property values on the participation decision to verify that the individual-level simulated behavior matches the data. Chetty et al. (2017) (Section D) estimate

\[
\Delta part_i = \alpha + \beta_1 \Delta PropertyValue_i + \beta_2 \Delta TotalWealth_i + \gamma \Delta X_{it} + \Delta \epsilon_i, \quad (16)
\]
Figure 5: Entry and Exit from Stock Markets - Data and Simulation

These figures plot the two-year entry/exit rates in stock markets in the data (solid line) and the model (dashed line).

where $\Delta x_i \equiv x_{i,t+1} - x_{i,t-1}$ for an individual who purchased a house in period $t$. The purpose of this regression is to control for selection while answering: “Do households who buy more expensive houses reduce their stock holdings by a larger amount from the year before to the year after home purchase?” To deal with the endogeneity in the size of a house one purchases, they instrument for property values using state house price indices. When I estimate equation (11) on the simulated panel, I follow their procedure as close as possible. For more details on their estimation I refer to section C in Chetty et al. (2017), while a detailed discussion of my replication using simulated data is in Appendix D.1 and Table 7 compares the estimation samples.

Table 5 include the result in Table 5 from Chetty et al. (2017) on participation, as well as the same regression in the model. Chetty et al. find that an increase in the property value of $\$100,000$ in 1990 dollars, keeping home equity and mortgage debt constant, decreases the probability of participation by 12 percentage points. The same increase in total wealth increases the portfolio weight by 5.88 percentage points. Overall, the estimates from the simulated panel are reported in Column (2) of Table 5 and match the effects found in Chetty et al. (2017) well. From these overall results, I conclude that the model not only matches average life-cycle moments well but also captures on an individual level the impacts of property values and total
Table 5: Replicating Chetty et al. (2017)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ Property Value</td>
<td>-12.14</td>
<td>-24.92</td>
</tr>
<tr>
<td>(× $100K)</td>
<td>(2.56)</td>
<td>(1.2)</td>
</tr>
<tr>
<td>∆ Total Wealth</td>
<td>6.10</td>
<td>7.3</td>
</tr>
<tr>
<td>(× $100K)</td>
<td>(0.96)</td>
<td>(1.58)</td>
</tr>
<tr>
<td>Observations</td>
<td>6912</td>
<td>60207</td>
</tr>
</tbody>
</table>

Note: Standard errors in parentheses. The results for specifications 1 is from column 5 in Table VI in Chetty et al. (2017). All specifications use a two-stage least squares estimator where the state price index instruments for the property value. The set of control variables is the same as in Chetty et al. (2017) except that the simulated model has no year fixed effects, and use the persistent human capital shock $v$ as a proxy for education.

6 Conclusion

This paper improves our understanding of the causes of the low stock market participation rate over the entire life cycle in the US. New salient empirical results highlight the high exit rate (about 20%) from stock markets as a reason for the low participation rate among young and middle-aged households and the relative importance of homeownership in driving exit decisions. Based on the empirical results, it develops a quantitative life-cycle portfolio choice model that can match the four main components of households’ portfolios over the life-cycle: the participation rate, homeownership rate, conditional portfolio weight, and net worth. To do so, the model requires a low participation cost and a high risk aversion parameter relative to a model without housing.

Adding housing to the benchmark models (e.g., Fagereng et al., 2017; Cocco, 2005) improves the model fit for participation over the entire life cycle. The main mechanisms are that liquidity constrained (‘home poor’ or ‘wealthy hand-to-mouth’) homeowners delay stock investment to build sufficient home equity. A model with a
one-time cost and housing (e.g. Vestman (2019); Cocco (2005)) eventually predicts 100% participation among middle-aged and/or retirees. This paper shows that a model with housing and per-period costs can match the participation rate over the entire life-cycle. The combination of per-period costs and house-tenure allows the model to not only match targeted moments but it also qualitatively matches the entry and exit rates over the life cycle. Additional to matching targeted and non-targeted life-cycle moments, regressions on the simulated households recover the causal impacts of home equity and property values found in Chetty et al. (2017).

References

Ameriks, J. and S. P. Zeldes (2004): “How Do Household Portfolio Shares Vary With Age?”.


A Supplementary Figures and Tables

Figure 6: Life Cycle Participation, Homeownership, Entry and Exit

(a) Joint Wealth and Ownership Distribution

(b) Joint Wealth and Ownership Distribution

B More Model Details

B.1 Renters’ and Buyers’ Decision Problems

B.1.1 The renter’s decision problem

Below I fully describe the decision problem facing an agent who doesn’t own a house at the beginning of the period. The renter chooses optimal consumption, rental size, (positive) bond holdings and if he chooses to pay the fixed cost $q$, also how much to save in stocks:

$$V_a^R(x, v, p) = \max_{c,b',s',h} \{ u(c, h) + \beta_a E_a[V_{a+1}(x', v', p', 0)] \}$$

$$x + w = c + 1(s' + q) + b' + fph$$  \hspace{1cm} (17)

$$x' = s'(1 + r') + b'(1 + r')$$

$${c, b', s'} \in [0, \infty)^3, h \in \{small, large\}$$

Processes for $w, r, p$ (eq. 7,5,6)
Table 6: Summary Statistics by Ownership in the PSID

<table>
<thead>
<tr>
<th></th>
<th>PSID Renters mean</th>
<th>PSID Homeowners mean</th>
<th>SCF Renters mean</th>
<th>SCF Homeowners mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial Portfolio</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock Participation</td>
<td>0.21</td>
<td>0.52</td>
<td>0.26</td>
<td>0.60</td>
</tr>
<tr>
<td>Exit</td>
<td>0.34</td>
<td>0.19</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Entry</td>
<td>0.10</td>
<td>0.23</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Weight on Stocks</td>
<td>.</td>
<td>.</td>
<td>0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>Cond. Weight on Stocks</td>
<td>.</td>
<td>.</td>
<td>0.49</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Financial Portfolio (w/o IRA)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock Participation</td>
<td>0.12</td>
<td>0.32</td>
<td>0.15</td>
<td>0.30</td>
</tr>
<tr>
<td>Exit</td>
<td>0.40</td>
<td>0.29</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Entry</td>
<td>0.05</td>
<td>0.12</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Weight on Stocks</td>
<td>0.04</td>
<td>0.07</td>
<td>0.02</td>
<td>0.04</td>
</tr>
<tr>
<td>Cond. Weight on Stocks</td>
<td>0.14</td>
<td>0.16</td>
<td>0.31</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Economic variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income (thousands)</td>
<td>46.05</td>
<td>99.42</td>
<td>44.23</td>
<td>118.57</td>
</tr>
<tr>
<td>Wealth (thousands)</td>
<td>57.72</td>
<td>540.33</td>
<td>82.59</td>
<td>816.21</td>
</tr>
<tr>
<td>Stocks (thousands)</td>
<td>10.18</td>
<td>84.43</td>
<td>2.41</td>
<td>18.36</td>
</tr>
<tr>
<td>House Value (thousands)</td>
<td>0.00</td>
<td>244.60</td>
<td>−0.00</td>
<td>279.48</td>
</tr>
<tr>
<td>Home Equity (thousands)</td>
<td>0.18</td>
<td>156.31</td>
<td>−0.00</td>
<td>186.36</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School</td>
<td>0.55</td>
<td>0.52</td>
<td>0.56</td>
<td>0.51</td>
</tr>
<tr>
<td>College</td>
<td>0.23</td>
<td>0.33</td>
<td>0.20</td>
<td>0.34</td>
</tr>
<tr>
<td>Married</td>
<td>0.26</td>
<td>0.67</td>
<td>0.39</td>
<td>0.69</td>
</tr>
<tr>
<td>White</td>
<td>0.82</td>
<td>0.92</td>
<td>0.56</td>
<td>0.79</td>
</tr>
<tr>
<td>Age</td>
<td>43.69</td>
<td>54.39</td>
<td>42.42</td>
<td>53.61</td>
</tr>
<tr>
<td>Family Size</td>
<td>1.97</td>
<td>2.55</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

*Note: Author’s own calculation from the PSID, waves 1984-2015. All observation are equally weighted. Exit and entry is relative to the previous observation, i.e. a five year gap for waves 84, 89, 94, 99 and two year gap thereafter.*
B.1.2 The buyer’s decision problem

Compared to renters and stayers, buyers decide on how big of a house to own, don’t pay rent, can borrow in the bond but also pay moving costs.

\[
V^B_a(x, v, p) = \max_{c, b', s', h'} \{u(c, h') + \beta_a \mathbb{E}_a[V^B_{a+1}(x', v', p', h')]\}
\]

\[
x + w = c + 1(s' + q) + b' + ph'(1 + mc)
\]

\[
x' = s'(1 + r_{a+1}) + b'(1 + r^f) + hp'(1 - \delta)
\]

\[
\{c, s'\} \in [0, \infty)^2, b' \geq -(1 - d)ph', h' \in \{small, large\}
\]

Processes for \(w, r, p\) (eq. 7,5,6)

\[
\text{B.2 Nesting models without housing}
\]

B.2.1 Nesting Fagereng, Gottlieb and Guiso (2017)

The model extends Fagereng et al. (2017) by including housing. To obtain their model one only must limit the economy to not include housing, i.e. setting \(h \in \mathcal{H} = \{0\}\). Under this assumptions the house price \(p\) is redundant, as is the discrete choice between renting, staying and buying a new house. The household decision problem is then simplified into

\[
V^{FGG}_a(x, v) = \max_{c, b', s'} \{u(c) + \beta_a \mathbb{E}_a[V^{FGG}_{a+1}(x', v')]\}
\]

\[
x + w = c + 1(s' + q) + b'
\]

\[
x' = s'(1 + r^f) + b'(1 + r^f)
\]

\[
\{c, b', s'\} \in [0, \infty)^3
\]

Processes for \(w, r\) (eq. 7,5)
B.2.2 Nesting Cocco, Gomes and Maenhout (2005)

To nest Cocco et al. (2005) only two further assumption is required; that the fixed cost is zero \((q = 0)\) and the probability of a tail event is equal to zero \((p_{\text{tail}} = 0)\). This removes one discrete choice (whether to participate), and mechanically implies 100% stock market participation.

\[
V_{CGM}^a(x, v) = \max_{c', b', s'} \left\{ u(c) + \beta_a \mathbb{E}_a[V_{a+1}(x', v')], \right. \\
x + w = c + s' + b' \\
x' = s'(1 + r') + b'(1 + r_f) \\
\left\{c, b', s'\right\} \in [0, \infty)^3
\]

(20)

Processes for \(w, r\) with \(p_{\text{tail}} = 0\) \((\text{eq. 7,5})\)

C Numerical Details

C.1 Solution Algorithm

The problem is solved backwards, by first solving the value function of a retiree at age \(T\), when death is certain. In the final period households liquidate all wealth for goods consumption and hold no housing wealth. This process is repeated backwards, until age \(a = 25\). All stochastic elements are discretized following Rouwenhorst (1995). The persistent income shock \(v\) follows a three-state Markov chain process, and the transitory income shock is discretized to three states. The house price shock \(\varepsilon^h\) is discretized to five states. The stock market return shock is discretized to have six states (five normal returns and the tail return). The net worth \(x\) and price \(p\) grids are both unevenly spaced, with higher density for lower values. For values of \(x\) and \(p\) not on the grids I use linear interpolation.
C.2 Details on Moment Construction

I use the following procedure to estimate all the four life-cycle moments. I first group households into 5-year bins from 25-29 to 80-84. Next I drop all households who are in the top or bottom percentile of wealth within each age group. I then reweigh all sample weights to have constant sum of weights across years. I then run weighted linear regressions of the moment on the age dummies and year dummies. I then predict the value of the moment for each age × year, and then find the average over years to obtain the life-cycle profile. When calculating the simulated moments there are no year effects so I simply find the average within each age group.

C.3 Structural Estimation

The estimation procedure is similar to the one in Daruich (2018):

1. Outside Estimation: I first set parameters that pin down the stochastic processes and prices outside the model. These parameters are either estimated directly from the data or set to standard values in the literature. These parameters are listed in Table 3.

2. Global Search: As there is no reason to think that the objective function will have a single local minimum, I use a global optimization procedure. First, I draw 10,000 parameter vectors ω from a six-dimensional hypercube (one dimension for each parameter) using a quasi-random low-discrepancy Sobol sequence. I then calculate the objective function for each parameter vector. The hope is that the parameter vector that minimizes the objective function approximates the global minimum.

3. I then perform a local search using a standard downhill simplex (Nelder-Mead), using as an initial guess the optimizer identified in the previous step. The convergence criteria is set to be that the percentage change between the objective
function evaluated at the worst and best parameter vector in the simplex is less than 0.1%.

D Details on Post Estimation Procedures

D.1 Sample Construction for Chetty et al (2017)

I now describe in detail how I prepare the simulated sample in order to replicate the empirical participation regression in Chetty et al. (2017). Parentheses denote the value or method used in Chetty et al. (2017). I define an indicator function for when households buys a house (renting to owning as well as owning to a new owned house). I then renormalize all dollar variables to be in $100,000. I then keep all observations the year before, during and after a house transaction. The participation indicator is multiplied by 100 to make it into a percentage. I keep all households aged 25-79. I then run IV regressions of the participation decision on property values (instrumented by the house price at the time of purchase), gross wealth, and a full set of indicators for age, productivity $v$ (‘education’) and economy (‘state’), equation (16).

D.2 Comparing Samples

Table r7 compares the descriptive statistics of the estimation sample in Chetty et al. (2017) and the simulated panel. Arguably the two samples are relatively close, but the model sample is somewhat wealthier and invests more heavily in stocks.

E Robustness Exercises

E.1 Changing the Down Payment Requirement

For young households who want to become homeowners, a key constraint is the down payment constraint. One of the main mechanisms in this model is that down
Table 7: Replicating Chetty et al. (2017)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>43.53</td>
<td>40.00</td>
<td>13.70</td>
<td>44.21</td>
<td>42.00</td>
<td>13.11</td>
</tr>
<tr>
<td>Income</td>
<td>53.13</td>
<td>42.98</td>
<td>46.86</td>
<td>56.63</td>
<td>52.47</td>
<td>22.28</td>
</tr>
<tr>
<td>Property Value</td>
<td>133.87</td>
<td>109.83</td>
<td>96.23</td>
<td>156.66</td>
<td>141.94</td>
<td>52.74</td>
</tr>
<tr>
<td>Mortgage</td>
<td>79.24</td>
<td>70.75</td>
<td>62.07</td>
<td>30.96</td>
<td>34.48</td>
<td>23.36</td>
</tr>
<tr>
<td>Gross Wealth</td>
<td>139.89</td>
<td>70.87</td>
<td>200.63</td>
<td>175.67</td>
<td>131.16</td>
<td>118.04</td>
</tr>
<tr>
<td>Liquid Wealth</td>
<td>35.46</td>
<td>5.57</td>
<td>98.33</td>
<td>25.77</td>
<td>0.00</td>
<td>62.24</td>
</tr>
<tr>
<td>Home Equity</td>
<td>54.99</td>
<td>31.38</td>
<td>73.04</td>
<td>125.70</td>
<td>106.23</td>
<td>64.76</td>
</tr>
<tr>
<td>Participation Rate</td>
<td>36.31</td>
<td>0.00</td>
<td>48.09</td>
<td>17.52</td>
<td>0.00</td>
<td>38.01</td>
</tr>
<tr>
<td>Portfolio Weight</td>
<td>22.52</td>
<td>0.00</td>
<td>35.27</td>
<td>43.01</td>
<td>39.15</td>
<td>36.25</td>
</tr>
<tr>
<td>Observations</td>
<td>6912</td>
<td>6912</td>
<td>6912</td>
<td>59363</td>
<td>59363</td>
<td>59363</td>
</tr>
</tbody>
</table>

Note: The descriptive statistics in the first two columns are taken from Table IIb in Chetty et al. (2017, p.1184). All variables in thousands 1990 US Dollars. Gross wealth in the simulation is defined as net worth plus mortgage \( (x + \text{abs}(b) \times 1_{\{b<0\}}) \) while liquid wealth is stocks plus bonds \( (s + b \times 1_{\{b>0\}}) \). The portfolio weight is defined as stocks over liquid wealth.

Payments generate non-monotone participation decisions in wealth. To shed light on how the down payment affect households’ portfolio decisions, I solve and simulate the model under minimum down payment requirements ranging from 0-100% using the estimated parameters. The simulated homeownership, participation, and conditional portfolio weights are plotted in Figure 7. Since the model has no market clearing or price adjustment and so any price effects and equilibrium effects that could arise from a change in the down payment are ignored.

Homeownership is decreasing in the down payment, and higher requirements progressively delay homeownership. However, the participation rate among the young is increasing in tightening borrowing constraints. The reason is two-fold. First, the participation among renters is increasing since a) their investment time horizon is longer, and b) they have more wealth and savings. Second, homeowners are also more likely to participate since they now have lower leverage and debt, which allows them to participate in the stock market sooner. The conditional portfolio weight responds little to changes in the down payment. However, we see that with higher down payments, we have higher participation, which effectively means that the mean...
Figure 7: Effects of the Minimum Down Payment

These figures plot the effect of increasing the size of the minimum downpayment. Darker, thicker lines have lower minimum downpayment ($d$) requirements.

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participant has lower wealth, which increases the average portfolio weight.

Finally, Figure 8 also plots the effects for the entry and exit rates as well as net-worth. Higher downpayments increase the entry rate over the entire life-cycle because of later entry to ownership for young and middle-aged and that downsizing now increases liquid wealth more for older households. The exit rate is lower only for younger households, indicating that renters now stay in the market for longer, and less new owners exit the stock market. Net worth is not affected by downpayments when prices are constant, so these effects are driven purely by the effects of the borrowing constraint on the portfolio choices.

E.2 Participation Rates by House Tenure

A major contribution in Vestman (2019) is to build a model that can rationalize the participation gap between renters and participants. In Vestman’s model, the main mechanism is preference heterogeneity (higher risk-aversion, lower discount factors, and higher participation costs) between a renter and owner type. I plot the participation rates by house tenure in Figure 9. We see that the model generally does worse at predicting the participation rate among renters than owner. However, the model correctly predicts the main feature of the data: that homeowners have
Figure 8: Effects of the Minimum Down Payment on Entry/Exit

These figures plot the effect of increasing the size of the minimum downpayment. Darker, thicker lines have lower minimum downpayment ($d$) requirements.

Figure 9: Participation By Homeownership - Data and Simulation

higher participation rates than owners.